Application of MHz frame rate, high dynamic range PIV to a high-temperature, shock-containing jet

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An experimental investigation is described for the study of a high-temperature, shock-containing jet emanating from a conic-section, converging-diverging nozzle. Particle image velocimetry (PIV) measurements were acquired for a flow field centered axially at the end of the jet potential core and radially along the lower half of the shear layer. For all cases the nozzle was operated at over-expanded conditions, and PIV images were acquired through the combined use of a pulse burst laser and a high-speed, gated intensified CCD framing camera. Because the system could acquire sequences of 16 images at MHz frame rates, temporally resolved measurements were able to be obtained. Each component of the unique PIV system is described in detail along with the experimental setup. In addition, a computational procedure developed for high dynamic range (HDR) analysis is presented with accompanying sample results. Estimations of the measurement errors associated with these results are also given. Finally, steps for improving the quality of the experimental data as well as the analysis procedure are offered as suggestions for future investigations.

I. Introduction

The work presented in this paper is part of a larger, collaborative effort to investigate the turbulence associated with jet noise generation. In addition to particle image velocimetry (PIV), synchronous near-field and far-field acoustic measurements were obtained to better quantify the noise generated by supersonic flows emanating from converging-diverging (C-D) nozzles. Such noise is associated with typical variable-area nozzles found on modern, high-performance, military aircrafts and consequently is of interest due to concerns over noise-induced hearing loss as well as degraded operational awareness. The cumulative data from these studies provides temporally resolved, synchronous characterization of the near-nozzle velocity field, the hydrodynamic pressure field, and the acoustic far field. By studying the noise-generating features of high-temperature, shock-containing jets using several measurement techniques, the hope is that a better understanding of near-nozzle flow conditions and their impacts on jet noise radiation can be obtained.

The following sections provide a comprehensive overview of the experimental research that was performed for this paper, that is, the acquisition of time-resolved (TR) PIV data for the near-nozzle flow field encompassing the collapse of the jet potential core (descriptions and preliminary results for the near-field and far-field acoustic investigations are available in Murray et al.†). The aeroacoustic motivation for this work is provided by introducing the problem of jet noise. Following such background information, accompanying sections are given that depict the experimental setup and the corresponding, operational parameters. The specialized facility where measurements were performed is described in conjunction with the unique PIV system that was used. It should be noted that results obtained for this experimental investigation will be presented at a later time. The aim of this paper is to discuss the development of a high dynamic range (HDR) processing scheme that will supplement conventional forms of PIV data analysis. The proposed HDR algorithm and its validation using synthetic particle images are presented in section IV. Steps for improving this HDR procedure are offered in the final section along with other concluding remarks.

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II. Experimental overview

Despite over 60 years of research in jet aeroacoustics, a limited understanding persists in regards to turbulent jets and the mechanisms responsible for turbulent noise generation. Not only has a universal definition of turbulent noise sources eluded theorists, but what constitutes a source mechanism and how such mechanisms can be rendered less efficient (as it pertains to sound generation) remain unknown. From an experimental standpoint attempts to model, measure, and control turbulent jets have been thwarted by instrumental constraints. More often than not, such constraints have resulted from inadequate sensitivities and insufficient frequencies to accurately capture or noticeably influence phenomena of interest. Until a better understanding of source mechanisms is achieved, efforts designed to eliminate or even minimize jet noise radiation will continue to be at the forefront of aeroacoustic research.

For the purposes of this paper in connection with the Jet Noise Reduction (JNR) program of the Office of Naval Research (ONR), jet noise corresponds to the high-amplitude sound generated by air-breathing propulsion systems, namely low-bypass turbine engines. At present it represents one of the most acute noise sources for the Department of Navy and has been linked to adverse biological, mechanical, and environmental effects. Such effects include but are not limited to the noise-induced hearing loss of Navy personnel, the structural degradation of Naval airframes, and the restriction of maintenance, testing, and training schedules due to noise pollution in communities surrounding Naval installations. To counteract these issues and combat the problem of jet noise, attempts are being made to realize and establish jet noise reduction technologies through coordinated science efforts. The multitude of experiments undertaken in this investigation represents one such effort. As mentioned, while the overall project goal is to obtain a benchmark-quality data set that includes time-dependent, velocity-field measurements along with synchronized near-field and far-field pressure signals, the research presented in this paper is solely concerned with the characterization of the near-nozzle velocity field.

II.A. Anechoic Jet Laboratory

Experiments were conducted in the Anechoic Jet Laboratory (AJL) at the Jamie Whitten National Center for Physical Acoustics (NCPA) on the campus of the University of Mississippi. The AJL is a small facility purpose built for the study of high-temperature, supersonic jet noise. To overcome the shortcomings of previous facilities, specifically the NASA Langley Small Anechoic Jet Facility (SAJF), the AJL is designed to allow for aspiration of the test chamber. Using a 10,000 standard cubic feet per minute (SCFM) fan, ambient air can be pulled through the facility at speeds of approximately 1 ft/s (measured without jet flow in the anechoic section). Because of upstream and downstream stagnation chambers, the air actually percolates into the 19-by-20-by-8 ft test chamber (measured between the wedge tips) through 50% porosity sliding panels in the upstream and downstream walls. This mode of operation results in a very even temperature distribution throughout the room while maintaining an acoustically anechoic environment. By aspirating the entire chamber, problems associated with localized heating can be minimized along with adverse effects on the jet entrainment due to the enclosed space. Figure 1 provides a view of the test chamber in the AJL with various measurement systems in place.

The jet rig visible in figure 1 and shown specifically in figure 2 is supplied air from an 1100 hp Ingersoll-Rand Centac compressor through a desiccant dryer system. A maximum volumetric flow rate of 5000 SCFM of dry (−40°F) air at 125 psia enables continuous operation of the facility at desired test conditions. Control valves operated in a closed-loop system allow the exit Mach number to be maintained within 1% of a specified value. Heat can be added to the flow through the use of a gaseous propane burner system as shown in figure 2(b). The actual propane combustor is housed well upstream of the nozzle assembly and is followed by a ceramic flow straightener and settling chamber. Although multiple nozzle assemblies exist for this system, only the configuration shown in the schematic that includes the centerbody section was utilized for the work in this paper. Investigations to characterize the effects produced by the other nozzle assemblies are currently underway. The compilation of results for each of these configurations will be presented in a future work.
Figure 1. Experimental setup in the test chamber of the Anechoic Jet Laboratory (AJL).

(a) Jet rig in the AJL

(b) Jet rig schematic (Murray et al.1)

Figure 2. AJL propane burner system and nozzle assembly.
II.B. MHz frame rate PIV system

A MHz frame rate PIV system was developed through the combined use of a pulse burst laser and a high-speed, gated intensified CCD framing camera. For its ability to acquire sequences of 16 images at MHz frame rates, the system allowed temporally resolved velocity-field measurements to be obtained for a high-temperature, supersonic jet. Each component of the unique PIV system is explained in detail along with the experimental setup. Although this system was synchronized with both near-field and a far-field pressure measurement devices, only the PIV system is considered for the purposes of this paper.

II.B.1. Pulse burst laser

As has been described in previous publications\(^4\)–\(^6\), a pulse burst laser system developed at Auburn University allows a specified number of high-energy, MHz rate laser pulses to be formed for a given burst of low-energy, short-duration pulses. It should be noted that several upgrades have been made to this system since these publications, including a new JDSU NPRO 126 continuous-wave (CW) Nd:YAG laser to enhance the pulse-to-pulse stability of each burst. This component in particular results in more consistent illuminations between images and thus better results in the PIV cross-correlations. In addition to the CW laser, three supplementary amplification stages have been incorporated into the system (for a total of six amplification stages) to increase the overall energy available for each burst and consequently each individual laser pulse. A schematic of the upgraded pulse burst laser system is shown in figure 3.

![Figure 3. Pulse burst laser system utilized for time-resolved (TR) PIV (top view).](image-url)

The design of the pulse burst laser system can be divided into three fundamental parts as indicated in the schematic: the pulse generation, the pulse energy amplification, and the frequency conversion. As the name suggests, the function of the pulse generation stage is to slice the output of the CW laser into a burst of low-energy, short-duration pulses. Slicing is achieved through the use of an acousto-optic modulator (AOM) that relies on the principles of the acousto-optic (AO) effect. In particular, a piezoelectric transducer is used to produce acoustic waves inside an optical crystal such that the traveling waves cause variations in the index of refraction of the crystal. To an optical beam, these variations appear as a sinusoidal grating in which the wavelength is equal to the acoustic wavelength. By controlling when and how frequently acoustic waves are produced inside the crystal, the generation of a specified number of short-duration pulses is possible depending on how often the CW input beam is disturbed. As with most AO devices, the AOM operates in
the Bragg regime where most of the incident light can be diffracted into the first-order beam fairly efficiently. This diffracted beam constitutes the desired burst of pulses utilized in experimental applications.

Following the formation of low-energy (nanojoule order), short-duration pulses, the remaining stages of the pulse burst laser consist of pulse energy amplification and frequency conversion. Amplification is provided by six flashlamp-pumped Nd:YAG rod amplifiers of increasing diameter and is necessary if the pulses are to be used for fluid-mechanical measurements. The first three amplifiers are used in a double-pass arrangement, whereas the final three allow only for single passes. Without going into detail, wave plates and polarizers provide the necessary means for achieving double passes through the first three amplifiers. Optical isolators between each of the first five amplification stages prevent problems associated with parasitic lasing and amplified spontaneous emission (ASE). By the end of the amplification chain, pulse energies have increased by a factor of more than $10^7$–$10^8$ and generally reach levels in excess of 50 mJ/pulse.

The final stage of the pulse burst laser system is the conversion of the beam’s wavelength from 1064 nm to 532 nm. This conversion is achieved via a nonlinear process inside a KTP crystal and results in an unavoidable loss of pulse energy. Nevertheless the beam, now in the visible spectrum, can be used for fluid-mechanical measurements including PIV and other flow visualization applications.

### II.B.2. Cordin 222-4G high-speed camera

Images are acquired using a Cordin 222-4G gated intensified CCD framing camera that is capable of recording 16 images at a maximum, equally spaced rate of 2,500,000 frames per second\(^b\). Such images are captured with a 2048 × 2048 px\(^2\) resolution, although the true resolution is less due to the inherent intensification process. The camera is able to achieve extremely high acquisition rates because it contains eight independently controlled optical pathways, each incorporating a microchannel plate (MCP) for signal intensification and ultimately terminating with a Kodak KAI-4022 CCD sensor. Schematics of the camera, including an interior view that illustrates four of the optical pathways, are shown in figure 4. By allowing each CCD to record 2 images, 16 total images can be acquired over a user-specified time period. Furthermore, because each pathway is independently operated, temporal spacing between frames is variable and can be set in an asynchronous fashion. Such flexibility even allows eight simultaneous exposures to be made. This feature is desirable since it enables eight theoretically identical particle images to be obtained, with any differences being directly attributable to error. More discussion on this topic will be given in the follow-up paper that includes the experimental results. For this work it is sufficient to note that because the camera can acquire 16 images over a user-specified, extremely short time period, temporal resolution is possible for all captured fluid motions. Additionally, the ability to obtain several particle images at varying time intervals relative to one another has provided the means of performing HDR PIV. Such measurements offer significant improvements over conventional PIV results since optimum temporal separations can be selected for different particle locations depending on the local velocity.

![Side profiles of the Cordin 222-4G camera utilized in the TR PIV investigation.](image)

\(^a\)This value is measured after the frequency conversion stage and thus accounts for the loss in energy associated with doubling the frequency of the Nd:YAG laser beam via a KTP crystal.

\(^b\)This rate assumes a necessary CCD transfer time of 3.2 µs (specified by Cordin) to ensure that the second exposure does not include images from the first exposure.

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5 of 21

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II.C. Experimental arrangement

The combined use of the pulse burst laser and the Cordin high-speed camera allowed TR PIV measurements to be made on a high-temperature (1350°F), supersonic jet. For all cases only a smooth bore C-D nozzle with an upstream centerbody section (shown in figures 5(a) and 5(b)) was considered at over-expanded conditions (Mach 1.55). Such conditions are typical of aircraft exhaust during takeoff and low-altitude operation. The actual nozzle consisted of two conic sections, one contracting and the other expanding, joined together to form a supersonic nozzle with a very sharp radius of curvature at the throat. This near discontinuity at the throat is significant since it allows shocks to exist even when the nozzle is operated at fully expanded conditions (Mach 1.74). To illustrate this effect, a mean profile of the near-nozzle velocity field determined by CFD is shown in figure 5(c). As indicated in the profile, the streamlined centerbody section was positioned well upstream of the nozzle contraction. It should be noted that this nozzle assembly without the centerbody piece represents a 1/10th scale model of the military power setting for the General Electric F414 engine.

![Nozzle centerbody (upstream view)](image1)
![Nozzle centerbody (downstream view)](image2)

![Mean velocity field determined by CFD for the centerbody configuration at fully expanded conditions (Murray et al.)](image3)

Figure 5. The centerbody section included in the nozzle assembly is shown in (a) and (b). The shock structures existing at fully expanded conditions are evident in (c).

The field of view for this work was chosen along the bottom shear layer of the jet and was centered at a distance 14 inches (7 jet diameters) downstream of the nozzle exit. This distance was selected based on previous measurements indicating the collapse of the jet potential core. The region imaged was slightly less than 16 square inches and was illuminated by a laser sheet directed vertically upwards and spanning axially along the centerline of the jet. This particular orientation was chosen for a variety of reasons including both the need to minimize disruptions in the anechoic environment as well as to ensure the most direct observation...
of any shear layer without passing the light sheet through the jet prior to imaging. This last point was especially important to prevent problems associated with aero-optical distortions. Figure 6 illustrates the position of the imaged region relative to the nozzle exit and the jet centerline (drawing not to scale).

![Figure 6. Position of the imaged region in the TR PIV investigation relative to the nozzle exit plane and the jet centerline (side view). The square region indicates the camera’s field of view.](image)

Figure 6. Position of the imaged region in the TR PIV investigation relative to the nozzle exit plane and the jet centerline (side view). The square region indicates the camera’s field of view.

Particle seeding for light scattering was achieved using aluminum oxide (Al$_2$O$_3$) particles nominally 0.1 µm in diameter. A nitrogen-pressurized reservoir filled with these particles was connected to the burner system upstream of the nozzle assembly and immediately following the propane combustor and diffuser, respectively. Four seeding tubes were attached around the burner system symmetrically to provide a uniform seeding density throughout the jet. To alleviate particle clumping, a spinning propeller inside the reservoir formed a cloud of aluminum oxide particles that was subsequently dispersed into the particle seeding tubes. Each connection between the reservoir and a tube was made in the supersonic portion of a miniature de Laval nozzle located at the entrance to each tube. This arrangement ensured that any surviving particle clumps were sheared apart before being injected into the burner system.

A schematic of the experimental setup is shown in figure 7. The 532 nm wavelength beam from the pulse burst laser was passed into the anechoic chamber perpendicularly to the jet axis and opposite the location of
particle displacements. Mathematically these relationships can be written as follows.

\[ u_{\text{max}} = \frac{\Delta x_{\text{max}}}{\sigma_{\Delta x}} \]  

In equation 1, the dynamic velocity range is represented by DVR, and the maximum and minimum resolvable particle displacements are shown as \( \Delta x_{\text{max}} \) and \( \sigma_{\Delta x} \), respectively. It should be noted that only velocity magnitudes are assumed in this definition. Thus if negative velocities occur, then \( u_{\text{max}} \) is defined as the larger of the maximum positive velocity or the maximum magnitude of the negative velocity. At present a DVR of approximately 200:1 is the standard for two-dimensional PIV measurements. In applications where ranges begin to exceed this value, the accuracy of vectors in the low-velocity regions starts to deteriorate. For this reason data acquisition rates are generally chosen such that only flow phenomena of interest are properly sampled. Unfortunately this mode of imaging means that vectors calculated in other flow regimes have the potential of being highly inaccurate. Consequently high dynamic range techniques have been developed to improve the evaluation of temporally resolved image sequences.

Although increasing the time interval between subsequent frames is not a preferred method for improving the DVR, a few studies have managed to achieve satisfactory results by applying this approach locally. The problem with using larger temporal separations is the reduction one experiences in the signal-to-noise ratio. A greater pulse separation leads to increased losses between in-plane and out-of-plane particle pairs. To
overcome this limitation, procedures known as multi-frame (MF) PIV have been developed that utilize temporally resolved, single-frame image sequences to locally optimize particle-image displacements.

One of the first MF methods designed to improve the performance of TR PIV is suggested by Fincham and Delerce. The approach revolves around a hierarchical processing scheme that considers the effects of local fluid deformation calculated during successive passes. Following an initial correlation of two frames with a temporal separation of $\delta t$, displacement estimates are made for the deformation and correlation of frames separated by larger time intervals ($2\delta t, 3\delta t, \text{etc.}$). By utilizing larger time intervals in subsequent correlations, average pixel displacements are increased and thus the overall DVR enhanced.

Persoons and O’Donovan propose a further development of this approach by taking higher-order effects into account. For this case images that symmetrically straddle a shared, central frame are considered. By locally optimizing the particle-image displacements, an optimum temporal separation can be chosen for each interrogation window that minimizes the relative measurement error.

A final method designed to enhance the precision and robustness of TR PIV measurements is introduced by Sciacchitano et al. The technique, referred to as adaptive multi-step ensemble correlation (AMEC), includes aspects of the MF approaches described previously as well as a method known as correlation ensemble averaging. For a short series of recordings separated by a constant time interval, optimum temporal separations are locally evaluated based on error-minimization criteria. Correlation signals acquired at different temporal spacings are linearly combined through the use of homothetic transformations. From comparisons with state-of-the-art PIV processing techniques, the AMEC method has proven to increase the reliability of measured vectors and to significantly reduce both precision and acceleration errors.

Until recently it has not been practical to use MF methods in high-speed flows due to limitations imposed by laser and camera repetition rates. To avoid the use of excessive temporal separations in TR PIV applications, a multiple pulse separation (MPS) technique is proposed by Persoons and O’Donovan. In this technique a series of double-frame images with different pulse separations is recorded such that a sequence with the following temporal distribution is obtained $\{[t, t + n_1\tau], [t + \delta t, t + \delta t + n_2\tau], \ldots\}$. The inter-frame time ($\delta t$) remains constant, whereas the pulse separation time ($\tau$) grows according to a monotonically increasing multiplier ($n_1, n_2, \text{etc.}$). Once a desired sequence has been acquired, vector fields for all pulse separation values are evaluated using standard PIV algorithms. A pulse separation optimality criterion is then applied locally to compute a final displacement field. Because the results encompass multiple pulse separation values, the DVR is dramatically increased compared to velocity fields achieved by conventional methods. Despite this improvement, the MPS technique applies only to mean flow and turbulence quantities since it is unable to provide temporally resolved results.

IV. Dynamic evaluation via ordinary least squares (DEVOLS)

As noted in the previous section, flows containing a wide velocity distribution present a major challenge to PIV algorithms. The reason is because particle motions in these flows (particle displacements in recorded images of these flows) can vary greatly depending on the local velocity. Since the entire range of flow velocities and thus particle motions cannot be adequately captured in a single interframe time, the temporal spacing in conventional PIV applications must be chosen such that only flow phenomena of interest are properly sampled. To overcome this problem and others related to temporal variations in particle-image patterns, HDR techniques like the ones described previously are currently being developed.

This section presents a novel HDR approach designed specifically for the experiments of interest, that is, the characterization of the near-nozzle velocity field in a supersonic jet using TR PIV measurements. The conceptual idea for the approach is based largely on the MF method developed by Hain and Kähler. Using the correlation results of symmetrically centered image pairs with increasing temporal separations, a single velocity field can be constructed entirely from local evaluations. This approach differs from the previous ones, however, in that individual vectors are determined from the combined influence of measurements achieved at multiple interframe times. By considering the results of several image pairs in each local evaluation, significant improvements can be made regarding measurement accuracy and individual vector quality. The following subsection discusses the proposed HDR processing scheme, termed dynamic evaluation via ordinary
least squares (DEVOLS), and its implementation into the experimental analysis. Validation for the procedure is given in the latter subsection by using synthetically generated images with known particle displacements. The effects that particle density and image noise have on the algorithm are specifically addressed.

IV.A. Proposed DEVOLS processing scheme

In the experiments of interest the ability to obtain 16 particle images at varying time intervals with respect to one another has provided the means of performing HDR PIV. Unlike conventional PIV where only one temporal spacing is available for all velocity determinations, the multiple combinations of image pairs in this investigation enable a single velocity field to be constructed from the results of several different local evaluations. In spatial regions where little or no particle motions are observed between subsequent frames, the results of image pairs spanning greater temporal distances are also considered. Thus it is entirely possible for vectors in the low-velocity regions of a flow field to be determined using the results of all available image pairs. In this way the DVR is dramatically improved because velocity ranges corresponding to a variety of interframe times are properly and simultaneously sampled.

A schematic illustrating the basic principle of HDR PIV in relation to the experimental investigation is shown in figure 8. As mentioned, sequences of 16 images were able to be obtained in which the \( \delta t \) between subsequent frames was specified to be 1 \( \mu s \). This particular interval was chosen such that the maximum flow velocities and their corresponding particle-image displacements would produce optimum results in the correlation analyses between consecutive frames. By setting the data acquisition rate to properly sample the maximum flow velocity, the slower velocity ranges were inherently oversampled and thus all image pairs could be used. Considering the temporal arrangement of all 16 frames in the schematic, the velocity field located between frames 8 and 9 is determined by considering the local results of all 8 image pairs symmetrically straddling the point designated \( t \). It should be noted that velocity fields at other points in time could also be computed, however this particular position allows the maximum number of image pairs to be used with central finite differencing. As is evident, for regions of the velocity field containing the highest local velocities, that is, the largest particle displacements between consecutive images, only the image pair shown in red is considered for analysis. Contrarily for regions containing little or no particle motions, multiple image pairs spanning larger temporal distances are considered. The image pair shown in blue represents the case of maximum temporal spacing and spans the entire sequence window of 15 \( \delta t \) or 15 \( \mu s \). By using the information available across all 16 frames instead of only consecutive images, significant improvements in desired flow measurements can be made. Revisiting the mathematical relation discussed previously, the reason is because the DVR is increased by a factor of 15 compared to conventional PIV methods. The following equations demonstrate
this effect by combining the individual correlation results for the minimum and maximum interframe times.

\[
\begin{align*}
\delta t : & \quad u_{\text{max},1} = \frac{\Delta x_{\text{max}}}{\delta t} \\
15 \delta t : & \quad u_{\text{max},15} = \frac{\Delta x_{\text{max}}}{15 \delta t}
\end{align*}
\]

Considering only the relations denoted by the rectangular boxes above (equations 2 and 5), an improved DVR is obtained.

\[
\text{DVR} = \frac{u_{\text{max},1}}{\sigma_{u,15}} = \left[ \frac{\Delta x_{\text{max}}}{\sigma_{\Delta x}} \right] = 15 \frac{\Delta x_{\text{max}}}{\sigma_{\Delta x}} \quad (6)
\]

In the DEVOLS processing scheme all eight symmetrically centered image pairs are correlated in an initial step. This step represents the most computationally expensive and time-consuming portion of the procedure since each correlation utilizes multiple passes along with several of the latest PIV processing techniques (window deformation and subpixel refinement schemes). To perform the correlation analyses, Dantec Dynamics software (DynamicStudio v3.31: Smart Software for Imaging Solutions) is used. In order for a final velocity field to be accurately constructed from local results, it is imperative that the same number of interrogation windows be used in each of the eight correlations. This requirement ensures that the vector spatial locations in each displacement field are positioned and scaled in a 1:1 ratio.

Following the eight correlation analyses, error-minimization criteria are employed to select the most accurate vectors determined at each spatial location. Such criteria are user-specified to allow for increased processing flexibility. To avoid errors stemming from particle accelerations, displacement measurements are restricted by a maximum displacement limit. This upper bound ensures that high velocity regions are only assessed by image pairs spanning the smallest temporal separations. Contrarily it enables low velocity regions to be evaluated by the maximum number of image pairs. In most cases the highest accuracies are achieved when particle displacements are required to satisfy the one-quarter rule. This rule suggests that in-plane displacements should not exceed one-quarter of the interrogation window size used in the correlation analyses.\(^{18}\) Although this rule is rendered obsolete by the use of multi-pass/multi-grid algorithms in the correlation analyses (except for the initial coarse grid), it provides a reasonable albeit rudimentary condition for the current processing scheme. As will be described below, the algorithm at present requires valid particle-image displacements to increase in a linear fashion over increasing temporal spacings. Modifications to the program in the near future will remove this assumption by inherently accounting for local acceleration effects. More on this topic is given in the concluding remarks section.

In addition to the particle displacement limit, a specified level of sensitivity is applied by the DEVOLS algorithm when considering the validity of vectors. This criterion is based on the notion that for negligible particle accelerations, all image pairs should in principle provide the same velocity measurement for a given spatial location. Because particle motions under a zero-acceleration condition remain constant with time, a linear trend is observed if their displacements are plotted over time. Considering only the measurements at a single spatial location satisfying the maximum displacement restriction, a linear trend should be observed if they are plotted against their corresponding interframe times. Therefore using ordinary least squares (OLS) statistics, a linear regression line can be fit to the data in which the slope is indicative of the local velocity. A measure of how well this regression line fits the displacement measurements is provided by the coefficient of determination, denoted R\(^2\). Depending on the user-specified sensitivity level, i.e., the minimum-allowable R\(^2\) value, displacement measurements with the largest residuals are simply rejected until either the R\(^2\) value exceeds the required tolerance or only a single, default measurement remains. The default measurement in the current algorithm corresponds to the particle displacement determined by the image pair spanning the shortest \(\delta t\). Thus the default velocity measurement for each spatial location represents the conventional PIV measurement associated with the overall data acquisition rate. In the experiments of interest this interval is the 1 \(\mu s\) separation time between consecutive, subsequent frames. Only the default value is used at each location unless supplementary measurements provided by the additional image pairs are deemed valid by the user-specified, error-minimization criteria.

To better explain the DEVOLS processing scheme, consider the plot shown in figure 9. In this example all eight displacement measurements for a given spatial location are plotted against their corresponding
interframe times. Because 32 × 32 px\(^2\) interrogation windows were specified during the correlation analyses, a maximum displacement limit was set at 12 pixels. This value is slightly larger than the one-quarter rule would suggest. Based on this restriction (depicted by the dashed red line in the plot), five of the eight measurements are immediately rejected. As mentioned, imposing a displacement restriction serves to minimize effects caused by particle accelerations. Such effects become increasingly pronounced over excessive particle displacements since the algorithm currently assumes only linear trends. Considering the remaining three measurements, a linear regression line is computed using OLS statistics with the added constraint that the line pass through the origin (since the limit of ∆x = 0 as δt → 0). Because the minimum R\(^2\) value was specified to be 0.975, the measurement with the largest residual (labeled Outlier in the plot) is rejected. After computing a second OLS regression line, the new R\(^2\) value is found to exceed the specified tolerance. Thus the slope of this final regression line represents the local velocity determined for this particular spatial location. The process is repeated for all spatial locations until a final velocity field is achieved.

Figure 9. Graphical explanation of the DEVOLS processing scheme applied at a single spatial location. Measurements are deemed invalid based on a maximum displacement limit of 12 pixels and a minimum R\(^2\) tolerance of 0.975 (both of these criteria are specified by the user). The slope of the final OLS regression line is indicative of the local velocity.

IV.B. DEVOLS validation

To validate the DEVOLS processing scheme described previously, a time-resolved sequence containing 16 synthetically generated particle images was considered. The particle density (ρ\(_{\text{img}}\)) in each image was chosen such that on average each interrogation window would contain 24 particles. The particle diameters were allowed to vary at most by 1 pixel from a nominally specified value of 5 pixels\(^c\). In addition, the particles were allowed to exit the field of view based solely on their in-plane motions. The particles could also vary in depth within the light sheet (sheet thickness for the range [0 1] was set at 0.3), although such positions were fixed since no out-of-plane motion was permitted. For this initial case only zero-noise conditions were simulated. To maintain consistency with the Cordin CCD sensors, the image sizes were specified to be 2048 × 2048 px\(^2\). It should be noted that the program used to generate this sequence was a modified version

\(^c\)Large particle-image diameters were specified to better resemble the particles recorded in the experimental images. As mentioned, the resolution in these images is inherently reduced due to the intensification process associated with the camera. Consequently the minimum resolvable particle diameter associated with this camera is slightly larger than would be expected for a non-intensified CCD camera with a comparable sensor size.
of the synthetic particle-image generator in PIVlab, the time-resolved digital particle image velocimetry tool for MATLAB.\textsuperscript{19}

The flow field simulated in the synthetic image sequence was a Hamel-Oseen vortex centered at the position (1024.5, 1024.5). The maximum circulation in terms of maximum particle displacement between consecutive, subsequent frames was limited to 8 pixels. This value was chosen such that the one-quarter rule would be satisfied for $32 \times 32$ px\textsuperscript{2} interrogation windows over a single interframe time. A 50\% overlap was specified for these windows during the correlation analyses. A vortex was chosen to validate the DEVOLS processing scheme because it provides a geometrically simple case of flow containing a wide velocity range. Depending on the interframe time, portions of the flow field are inherently under-sampled or drastically over-sampled when investigated by conventional PIV. This effect is evident in the individual correlation results presented in figure 10. A small $\delta t$ provided by the central image pair can resolve the high-velocity regions

![](image1.png)

Figure 10. Velocity fields determined by the image pairs spanning $\delta t$ and $15 \delta t$, respectively. Every third vector is shown for clarity since each field contains over 16,000 vectors. As is evident by the corresponding absolute error plots, in both cases a single interframe time is insufficient to resolve the full DVR of the flow. The contour color is indicative of the total deviation in pixels from the analytical solution.
near the vortex core but in doing cannot adequately resolve the remaining low-velocity regions (figure 10(a)). As a result, particle motions in these regions approach the limit of minimum resolvable displacement and thus the measurement accuracy for individual vectors is of poor quality. Contrarily a large interframe time provided by the image pair spanning the entire temporal window of $15\delta t$ is unable to resolve the core (figure 10(c)). In this case, however, the low-velocity regions surrounding the core and nearing the image edges are highly resolved. The contour plot presented alongside each vector plot corresponds to the total deviation, or absolute error, in pixels that exists for that particular measurement ($\delta t$ and $15\delta t$, respectively). The patterns visible in each plot illustrate the trends described previously. For both cases the low DVR in the measurements severely limits the viability of conventional PIV applications. It should be emphasized that because this investigation represents the ideal case of zero noise, the individual correlations perform very well. Still, the patterns visible in the absolute error plots clearly illustrate the effectiveness of the algorithm.

Using the DEVOLS method described previously, any number of displacement results from the eight correlation analyses can be considered at each spatial location to obtain a more accurate measurement of the velocity field. The fact that multiple interframe times are able to be used allows this measurement to characterize a much higher DVR of the flow than before. The plot shown in figure 11(a) contains the final HDR result. The error minimization criteria specified in the DEVOLS algorithm for this case were 12 pixels for the displacement limit and 0.975 for the $R^2$ tolerance. Unless specified, these values should be assumed for the HDR results contained in the remainder of this paper. As before, the total deviation in pixels from the analytical solution is shown in figure 11(b).

![Vector plot for the HDR result](image1.png)  
![Absolute error from the analytical result](image2.png)

**Figure 11.** Velocity field and absolute error for the HDR result, respectively. The HDR result characterizes the DVR of the flow much better than the results shown in figures 10(a) and 10(c) since it is not restricted to a single interframe time. The pattern visible in the contour plot clearly illustrates this fact.

From the plots above, the measurement accuracy in the HDR result is much better than in the results shown previously for a single interframe time. The obvious reason is because the DEVOLS processing scheme can locally utilize the displacement results of multiple image pairs and thus multiple interframe times to more accurately sample the variety of flow regimes. For this flow field only the image pair spanning the shortest $\delta t$ should ideally be used to sample the high-velocity regions near the vortex core. Contrarily multiple image pairs with successively increasing temporal separations should be used to sample flow regimes located at increasing spatial distances from the core. Considering the plot shown in figure 12(a), this exact trend is observed. The contour coloring scheme is indicative of the number of displacement measurements ($N_{vec}$) utilized by the DEVOLS algorithm to determine the final OLS regression line at each vector location. Because the minimum velocities in this flow field were still significantly higher than the minimum resolvable limit (assumed to be 0.1 pixels), only half of the image pairs were able to be used at any given location based on the specified settings (12 pixel displacement restriction and 0.975 $R^2$ tolerance). Figure 12(b) also illustrates this trend by plotting the velocity profile measured along the central, horizontal slice through...
the vortex. As is evident, the HDR result shown in red almost exactly matches the analytical solution, whereas the individual correlation results show significant deviations. The results derived from the image pairs spanning the largest temporal distances are especially ineffective at resolving the high-velocity regions near the core.

Figure 12. Results for the vector evaluation field as well as the central velocity profile, respectively. The trends observed in both cases indicate that only the image pair spanning the shortest temporal distance is capable of accurately resolving the high-velocity regions near the vortex core. Contrarily multiple image pairs spanning increasing temporal distances are able to resolve the low-velocity regions. Thus an increased number of displacement results can be utilized to determine the velocity vectors located at increasing spatial distances from the core.

IV.B.1. Varying particle density

To characterize the effect that different particle densities have on the DEVOLS algorithm, five temporally resolved image sequences were synthetically generated for the Hamel-Oseen vortex previously described. All parameters (particle diameter and variation, allowable depth within the light sheet, zero out-of-plane particle motion, maximum circulation or particle displacement between subsequent frames, and zero noise conditions) were held constant between the sequences except the total particle number. In each case this total particle number was chosen such that a desired, average number of particles would be found in the interrogation windows. Similarly as before, $32 \times 32$ px$^2$ windows were specified in the correlation analyses with a 50% overlap. The particle densities considered in this investigation and written in terms of particles per interrogation window are as follows: 3, 6, 12, 24, and 48. The HDR results shown previously for a particle density of 24 correspond to the same image sequence utilized in this investigation.

Considering the plot shown in figure 13, the particle densities chosen for this investigation did not appear to affect the DEVOLS algorithm in any appreciable way. A few blips are noticeable for $\rho_{img} = 3$, however the overall velocity profile still closely resembles the analytical result. One would expect the measurement error to increase for densities below 3 due to the lack of particles and consequent ambiguity in the correlation analyses. One would also expect the error to increase above 48 due to the increased overlapping of particle images in the recordings. Quantifying such levels where the DEVLOS processing scheme begins to experience significant errors relative to conventional PIV algorithms is certainly a topic warranting further investigation.
IV.B.2. Varying noise conditions

Two investigations were conducted to characterize the effects that different noise sources have on the DEVOLS algorithm. In both cases four image sequences of increasing noise levels were considered against a baseline sequence with zero noise. As before, a Hamel-Oseen vortex was simulated in the particle motions, and all parameters were held constant between sequences except for the variable in question. In both investigations the chosen particle density was 24 ($\rho_{\text{img}} = 24$), and care was taken to ensure that the initial positions of all particles remained the same. Thus all image sequences were identical in terms of particle density, particle distribution, and particle motion, however they differed in the prescribed level of noise.

Salt and pepper noise The first investigation was designed to simulate losses-of-pairs in the correlation analyses. This feat was accomplished by increasing the level of salt and pepper noise in the image sequences. Such noise affects the individual images within a sequence differently by randomly turning a number of pixels on to the maximum intensity value or off to the minimum intensity value. The number of pixels affected in each image is governed by a specified noise density ($\rho_{\text{noise}}$) applied to all of the images for a given sequence. Multiplying this value by the total number of pixels in each image provides the total number of pixels affected. Noise densities of 0, 0.025, 0.05, 0.075, and 0.1 were considered in this investigation. Sample particle images for $\rho_{\text{noise}} = 0$ (baseline) and $\rho_{\text{noise}} = 0.075$ are shown in figure 14, respectively.

The plot shown in figure 15 contains the HDR results of the five image sequences considered, namely the baseline case as well as the four cases of increasing noise. Similarly to before, this profile corresponds to the local velocity measurements along the central, horizontal slice through the vortex. As expected, the number of deviations from the analytical solution appears to increase with increasing noise density, although all results resemble the analytical solution rather closely. The fact that large deviations appear at different positions for different image sequences indicates that the salt and pepper noise applied to each sequence was sufficiently random. Thus the losses-of-pairs in the correlation analyses that resulted in the observed spikes were clearly due to the noise and not some artifact in the underlying particle-image distribution.

Figure 13. HDR results for the Hamel-Oseen vortex in which the particle density was varied. Five temporally resolved image sequences were synthetically generated to contain 3, 6, 12, 24, and 48 particles per interrogation window. The velocity profile shown corresponds to the central, horizontal slice through the vortex.
Figure 14. Image regions containing different levels of salt and pepper noise. Each region measures $128 \times 128$ px$^2$ and contains an identical particle density and particle distribution.

Figure 15. HDR results for the Hamel-Oseen vortex in which the level of salt and pepper noise was varied. Five temporally resolved image sequences were synthetically generated to contain noise densities of 0, 0.025, 0.05, 0.075, and 0.1 (where the density multiplied by the number of pixels per image yields the total number of pixels affected). The velocity profile shown corresponds to the central, horizontal slice through the vortex.
The plots shown in figure 16 correspond to the HDR result for \( \rho_{\text{noise}} = 0.075 \). Figure 16(a) shows the entire velocity profile, whereas figure 16(b) contains an enlarged view of the region indicated by the axes. Despite several wild deviations from the analytical solution by the individual correlation results, the HDR curve follows the analytical result rather closely. As expected, the correlations of image pairs with the largest temporal separations were mostly affected in the high-velocity regions. Thus large deviations are visible around the peaks just outside the central core. In contrast the image pair spanning a single \( \delta t \) was mostly affected in the low-velocity regions near the image edges. Although the observed deviations for this case are significantly less than those seen for the large interframe times, they are clearly visible in the zoomed view.

Gaussian white noise The second investigation was designed to simulate intensifier noise in the correlation analyses by increasing the level of Gaussian white noise in the image sequences. For clarity the term Gaussian refers to the distribution of noise values in each image (how frequently a particular value appears), and white describes the flat shape of the frequency spectrum. As such the noise applied to any two images (separated in time) for a given sequence is statistically independent or uncorrelated. Analogous to the noise density in the previous investigation, the mean (\( \mu \)) and variance (\( \sigma^2 \)) specified in the Gaussian distribution provided the means of controlling the noise level generated for each sequence. In all cases a zero-mean was specified, and variances of 0, 0.025, 0.05, 0.075, and 0.1 were considered. Sample particle images for \( \sigma^2 = 0 \) (baseline) and \( \sigma^2 = 0.075 \) are shown in figure 17, respectively.

Presenting all results in the same order as before, the plot shown in figure 18 contains the HDR results of the five image sequences considered, namely the baseline case as well as the four cases of increasing noise. Again this profile corresponds to the local velocity measurements achieved along the central, horizontal slice through the vortex. Analogous to the trend observed for increasing particle density, the number of deviations from the analytical solution appears to increase for increasing values of variance. Nevertheless all results resemble the analytical solution to some extent.

The plots shown in figure 19 correspond to the HDR result for \( \sigma^2 = 0.075 \). Figure 19(a) shows the entire velocity profile, whereas figure 19(b) contains an enlarged view of the region indicated by the axes. Although the HDR curve follows the general trend of the analytical solution rather closely, the deviations in this result are much more apparent than in the analogous HDR result shown for the previous investigation. In addition to the wild deviations experienced by the 15 \( \delta t \) and 11 \( \delta t \) correlation results near the core, small blips are evident in all of the individual results over much of the profile. Thus it appears that the Gaussian white noise, at least for the variances considered, had a much more profound effect on the individual correlation results and consequently the DEVOLS processing scheme. As is evident in the enlarged view, the HDR result for this case actually performed worse at several spatial locations compared to many of the individual correlations.
Figure 17. Image regions containing different levels of Gaussian white noise. Each region measures 128 × 128 px² and contains an identical particle density and particle distribution.

Figure 18. HDR results for the Hamel-Oseen vortex in which the level of Gaussian white noise was varied. Five temporally resolved image sequences were synthetically generated to contain zero-mean, Gaussian white noise with variances of 0, 0.025, 0.05, 0.075, and 0.1 (where the mean and variance were constant for all cases). The velocity profile shown corresponds to the central, horizontal slice through the vortex.
Figure 19. Results achieved for the central velocity profile by the image sequence with $\sigma^2 = 0.075$. Plots are shown that contain the entire velocity profile (a) as well as an enlarged view of the region enclosed by the specified axes values (b). The HDR result performed worse at several spatial locations compared to the individual correlation results.

The reason the HDR result in certain situations does not converge on the analytical solution, particularly in the low-velocity regions, is because the default measurement is always considered (the measurement provided by the image pair spanning $\delta t$ cannot be rejected in the current algorithm). This unfortunate scenario is the result of an inherent flaw that exists in the way the current processing scheme is designed. By not accounting for accelerations and thus assuming only linear displacements, it is possible in certain cases for the image pairs spanning the largest interframe times to provide a linear albeit incorrect slope for the particle-image displacement with time. Consequently if the OLS scheme is allowed to reject the default measurement in these cases, a regression line is fit to incorrect data. This situation is avoided in the majority of cases by forcing the algorithm to include the default measurement in all vector evaluations. Considering figure 19(b), however, this solution also results in the algorithm being significantly biased towards the default point. Work is currently being done to develop an additional criterion that will allow the default measurement to be rejected or ignored in the necessary cases. Two such methods are briefly discussed in the following concluding remarks.

V. Concluding remarks

The development and validation of a novel HDR processing scheme to supplement conventional PIV algorithms has been presented. As mentioned, this approach is currently being used to evaluate the TR PIV results from an experimental investigation regarding a high-temperature, shock-containing jet. The algorithm, termed dynamic evaluation via ordinary least squares (DEVOLS), offers substantial improvements over conventional PIV measurements for its ability to increase the dynamic velocity range. Unique to this approach is an iterative validation scheme than enables multiple displacement results to be utilized in the determination of an individual velocity vector. This feat is accomplished by fitting an ordinary least squares (OLS) regression line to those displacements satisfying a maximum displacement criterion at a given spatial location. The slope of this line is indicative of the local velocity. A user-specified tolerance, that is, a minimum allowable $R^2$ value, dictates how strictly this linear regression line must fit the data. To validate the DEVOLS algorithm, a temporally resolved sequence of synthetically generated particle images was considered in which the flow field surrounding a Hamel-Oseen vortex was simulated. Effects due to varying particle density as well as varying noise conditions in this flow environment were also characterized.

In addition to achieving results for the described experimental investigation, a couple of notable improvements to the DEVOLS algorithm are planned for the near future. The first involves transitioning the iterative validation scheme from an OLS regression model to a weighted least squares approach. This modification will allow increased emphasis to be placed on measurements corresponding to optimum particle displacements. Stated differently, instead of treating all valid displacement measurements equally, an additional criterion
will be used to determine the optimum temporal separation for a given spatial location. Increased weights will then be assigned to displacement measurements obtained by image pairs that span time intervals nearest this optimum value. In addition to this modification, the derivation of a least squares equation that accounts for acceleration is also in the works. This equation will enable regression lines to account for higher-order trends in the data and not be confined to linear fits. Such a scheme will also allow more data points to be considered at each location which will in turn improve the overall measurement accuracy.

References


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